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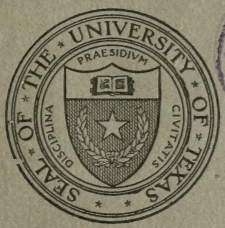

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The Texas Mathematics Teachers' Bulletin

Volume VIII, No. 3

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The benefits of education and of useful knowledge, generally diffused through a community, are essential to the preservation of a free government.

Sam Houston

Cultivated mind is the guardian genius of democracy. . . . It is the only dictator that freemen acknowledge and the only security that freemen desire.

Mirabeau B. Lamar

University of Texas Bulletin
No. 2320: May 22, 1923

The Texas Mathematics Teachers' Bulletin

Volume VIII, No. 3

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This Bulletin is open to the teachers of mathematics in Texas for the expression of their views. The editors assume no responsibility for statements of facts or opinions in articles not written by them.

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THE EQUIVALENCE OF EQUATIONS

In the process of solving equations in algebra we are accustomed to perform certain operations on the equations; for example, in solving the simple equation

$$2x+3=0,$$

we transpose the 3 to the other side and then divide by 2, and in more complicated equations or groups of equations a longer series of operations is required. This procedure raises the question as to what operations it is legitimate to use in solving an equation, and in particular what operations lead to new equations which have the same roots as the original ones. If two equations or two sets of equations have the same solutions, we say that they are *equivalent.* The usual process of solving an equation consists in reducing it to one or more simpler equations which are equivalent to the given one.

The operation of adding or subtracting terms on both sides of an equation, or transposing terms from one side to the other (which is the same thing), always leads to an equivalent equation; so does multiplication of both sides by a constant, or division of both sides by a constant other than zero. This follows directly from the axioms upon which the science of algebra is based, such as "If equals are added to equals, the sums are equal," etc.

Certain other frequently-used operations, however, may lead to new equations which are not equivalent to the given ones. For example, multiply both sides of the equation

$$2x=x+3$$

by x; the new equation

$$2x^2=x^2+3x$$

has two solutions, $x=0$ and $x=3$, whereas the given equation has only the solution 3. In general, multiplication by an expression involving the unknown is liable to introduce extra roots into an equation, namely the values of x which

make the multiplier zero. This operation is regularly used in the solution of a fractional equation; both sides are multiplied by the least common denominator of the fractions in order to reduce it to an integral equation. If care is taken to have all the fractions in their lowest terms, and if the multiplier is the least common denominator, the integral equation will usually be equivalent to the fractional one, but it is possible even if these precautions are observed to get an extra root, as the following example shows:

$$\frac{4x}{x^2-1} - \frac{x+1}{x-1} = 1.$$

Multiplying by the least common denominator x^2-1, we get the equation

$$4x-(x^2+2x+1)=x^2-1$$

or

$$x^2-x=0,$$

which has the roots 0 and 1, but only the first of these satisfies the fractional equation.

Another operation which may introduce extra roots is that of squaring both sides of an equation. For instance, the equation

$$2x=x+3$$

when squared becomes

$$4x^2=x^2+6x+9,$$

which is satisfied by $x=-1$ as well as by $x=3$. The reason for this is that the last equation is not merely the square of the given one, but also the square of

$$2x=-(x+3),$$

which has the root -1. In general, the equation obtained by squaring both sides of a given equation is satisfied both by the roots of the given equation and by those of the equation obtained by changing the sign of one side of the given equation.

The difficulty caused by these extraneous roots is not serious enough to lead us to avoid using the operations which introduce them. It merely becomes necessary to test all the values so obtained by substitution in the original

equation, and to reject those which do not satisfy it.

The method of squaring is regularly used in solving radical equations, such as

$$x=\sqrt{x+2};$$

here, as is customary, we understand the radical to denote the positive square root of $x+2$. If we square and transpose, we obtain the quadratic equation

$$x^2-x-2=0,$$

which has two roots, $x=2$ and $x=-1$. By substitution in the given equation we find that 2 is a solution but -1 is not. In such problems the student should be taught to regard the testing of the roots obtained as an essential part of the method of solution.

Conversely, certain operations lead to the loss of a root. If in the equation

$$(x-1)\ (x-3)=2(x-1)$$

we cancel the common factor $x-1$, we obtain the equation

$$x-3=2,$$

which is satisfied only by $x=5$, while the given equation has two roots, 5 and 1. The cancellation of a common factor is of course the same as dividing both sides of the equation by the common factor, and the root lost is a value of x which makes this factor zero. The process of dividing by a factor which contains the unknown should ordinarily be avoided in solving an equation; the proper way to solve the equation above is to transpose all the terms to one side and then factor:

$$(x-1)\ (x-3)-2(x-1)=0,$$
$$(x-1)\ (x-3-2)=0,$$
$$(x-1)\ (x-5)=0,$$

whence $x=1$ or 5. In this way we keep all the roots. Similarly, extracting the square root of both sides of an equation leads to the loss of a root, but the difficulty may be avoided in this case by using both square roots. Thus the equation

$$(x-2)^2=1$$

is equivalent to the two equations

$$x-2=1,\ x-2=-1.$$

In the case of two simultaneous equations in two unknowns, as in the case of a single equation, the process of solution involves replacing the given equations by an equivalent pair of simpler equations. Thus from the equations

$$2x+y=5,\ 2x-y=3,$$

we obtain by addition and subtraction the equivalent equations

$$4x=8,\ 2y=2,$$

from which the solution $x=2,\ y=1$ is at once obtained. Graphically, the two given equations represent straight lines intersecting at the point (2, 1), and in solving we replace them by a pair of lines through the same point parallel to the axes.

The same operations which are liable to produce non-equivalent equations when applied to single equations have the same effect when applied to sets of simultaneous equations. Thus if we multiply the equations

$$2x+y=5,\ 2x-y=3$$

by x and y respectively, we obtain the equations

$$x(2x+y)=5x,\ y(2x-y)=3y,$$

which have not one but four solutions, namely, $x=2,\ y=1$; $x=0,\ y=0$; $x=0,\ y=-3$; $x=2\frac{1}{2},\ y=0$.

The method of eliminating one unknown by adding or subtracting multiples of the given equations, which is used not only with simultaneous linear equations but also with simultaneous quadratic equations, is based on the following geometric theorem. Let u and v denote two algebraic expressions in x and y; then the curve represented by the equation

$$au+bv=0,$$

where a and b are constants, passes through the points of intersection of the curves $u=0$ and $v=0$. The proof of this is very simple. If (x, y) is one of the points of intersection of $u=0$ and $v=0$, then its coördinates must satisfy both of the equations $u=0$ and $v=0$; and if both u and v are equal to zero, obviously $au+bv$ is also equal to zero, so that the point (x, y) lies on the curve $au+bv=0$. As an example, consider the pair of linear equations above, and let

$$u=2x+y-5,\quad v=2x-y-3;$$

then the line

$$a(2x+y-5)+b(2x-y-3)=0$$

will pass through the intersection of $u=0$ and $v=0$. If we take first $a=1$, $b=1$ and then $a=1$, $b=-1$, we get the equations

$$4x-8=0,\ 2y-2=0,$$

which are equivalent to the ones obtained before by adding and subtracting the given equations.

A less simple example of the use of this theorem is furnished by the following pair of equations:

$$3x^2-2xy+y^2=6,$$
$$2x^2-xy+y^2=8.$$

Let

$$u=3x^2-2xy+y^2-6,$$
$$v=2x^2-xy+y^2-8,$$

and form the equation

$$4u-3v=0,$$

which is

$$6x^2-5xy+y^2=0,$$

or

$$(3x-y)(2x-y)=0.$$

This last equation is equivalent to the two equations

$$3x-y=0,\ 2x-y=0.$$

Since the curve $au+bv=0$ passes through the points of intersection of $u=0$ and $v=0$, we may replace either of the given equations by this pair of linear equations. Graphically, the given equations represent two ellipses, and we replace one of these by a pair of straight lines passing through their points of intersection and through the origin. The solution of the given pair of equations is thus reduced to the solution of two simpler pairs, namely

$$2x^2-xy+y^2=8,\ 3x-y=0$$

and

$$2x^2-xy+y^2=8,\ 2x-y=0.$$

The first pair has the solutions $x=1$, $y=3$ and $x=-1$, $y=-3$, the second pair has the solutions $x=\sqrt{2}$, $y=2\sqrt{2}$ and $x=-\sqrt{2}$, $y=-2\sqrt{2}$.

Any pair of simultaneous quadratic equations which contain no first degree terms can be solved in this way, by eliminating the constant term and then factoring the resulting equation. Other types of examples might be given, but those above are sufficient to display the principle involved.

P. M. BATCHELDER.

GEOMETRIC CONSTRUCTION BY MEANS OF THE COMPASSES ALONE

Teachers of plane geometry are familiar with the fact that certain geometrical constructions cannot be performed by means of the ruler and compasses. The most famous of these are the trisection of any angle and the squaring of the circle. In spite of the fact that definite proofs may be had of the impossibility of performing these constructions, tyros again and again offer solutions for these "perplexing and age-mystifying problems."

Another question of considerable interest is whether the construction of plane geometry performed by means of ruler and compasses can be performed by means of the compasses alone. The practical aspect gives added importance to this question when it is realized that the curve drawn by means of a set of compasses will be nearer to an ideal circle than the line drawn by means of a ruler is to an ideal straight line. Is it possible to dispense with the ruler and work with compasses alone?

An affirmitive answer was given by an Italian geometer, Lorenzo Mascheroni (1750-1800) over one hundred years ago. In a book entitled "Geometria del Compasso," published in Padua, 1797, he demonstrated that all the constructions of elementary plane geometry, which can be performed by means of ruler and compasses can be performed by means of the compasses alone. The lines in the problem are given by any two points on the line, a line segment by means of its two extremities. A point is determined as the intersection of two circular arcs (1), a line segment by its extremities and a line by two of its points. A construction requiring a line for its solution is therefore regarded as solved, if the determining points are found.

A simple example which conforms to the above requirements is the following: to construct the perpendicular bisector of a given line segment AB. This construction as given in our plane geometries is performed by means of the

compasses alone. It is substantially this, with A as center describe a circular arc of radius greater than ½ AB above and below AB. With the same radius and B as center describe arcs to intersect the first two. The two points of intersection C and D determine the required perpendicular bisector. This construction does not involve any fundamental steps other than finding the intersection of two circular arcs. The remaining fundamental steps are

(2) to find the intersection of a straight line and a circular arc, and

(3) to find the intersection of two straight lines. The steps (1), (2), (3), permit all of our constructions.

We may solve (2) and (3) with the compasses alone after we have established the following fundamental constructions:

I. Through a given point C, to draw a line parallel to a given line AB.

II. To double a given line segment AB.

III. To determine a point P′ which is symmetrical in a given line AB to a given point P.

IV. To bisect a given circular arc AB.

V. To find the fourth proportional to three given line segments m, n, p, respectively.

VI. To find the mid-point of a given line segment AB.

Construction I. Given the point C and the line AB, draw an arc with center at A and radius BC and with C as center and radius AB draw another arc. Then if D is their intersection on the same side as C of the line AB, ABCD is a parallelogram and CD is the required parallel to AB.

Construction II. Let AB be the given line segment. With A as center draw an arc of radius AB, also with B as center draw an arc of the same radius. Let one of the intersections of these be C. With C repeat the construction I above making use also of B. By two successive applications of construction I, two points D and E will be obtained lying on the same side of the line AB. CDE are in a straight line and CD=CE. Hence ED=2AB.

Construction III. Let P be the given point and AB the given line. If P is on AB, then P′=P. If P is not on AB, draw the arcs with centers at A and B and radii AP and BP, respectively. These will intersect in P′ on the other side of AB.

Construction IV. Let AB be the given arc with center at O. Determine the points D and E as in construction II. Let D correspond to A and E to B. With BD=AE draw arcs with centers at D and E, respectively, and let C be one of their intersections. Finally with OC as radius and D and E as centers, draw arcs to cut the arc AB. These will coincide in a point F which is the mid-point of arc AB.

Construction V. Let m, n, p be the given line segments. If $p \geqq 2m$, we may magnify both m and n in the same ratio, so that $p < 2m$. This can be done by successive applications of construction II. With any point as center draw two circles of radii kn and km where k is unity or a whole number to ensure that $p < 2m$. On the first circle strike off a chord AB=p. With A and B as centers and equal radii, draw arcs to intersect the other circle in C and D. Then the segment CE is the fourth proportional to m, n, p.

Construction VI. Let AB be the given line segment. By construction II we find the segment 2AB=AD. Now by the construction just given we find the point C such that AD:AB=AB:AC.

The proofs of constructions IV and V are excellent "originals" and are left to the reader to supply, as well as the necessary figures.

We may now return to the fundamental steps (2) and (3) above. (2) may be stated as the problem of finding the intersections of a given circle of radius r, and center, O, with a given line AB. If O is on the line AB, draw a circle with A as center to intersect the given circle in C and D. Then the required intersections are the two midpoints of the two arcs CE of the given circle. These are found by construction IV. If O is not on AB, we find by construction III, the point O′ which is symmetrical to O in the line AB. With O′ as center draw a circle of radius *r*.

This circle intersects the given circle in the required points.

Fundamental step (3) requires the intersection of the two lines AB and CD. Assume first that AB and CD are not perpendicular. We determine the points C′ and D′ which are symmetrical to C and D, respectively, in the line AB by construction III. The required point X will be on the line C′D′. By construction I we find the fourth vertex, E, of the parallelogram determined by C′D′D. Finally determine the point X such that CC′:CE=CD:Cg. Then X is the intersection of AB with CD. If AB and CD are perpendicular, then X is the mid-point of the segment CC′.

For further discussion of these problems and similar interesting questions the reader is referred to H. P. Hudson's Ruler and Compasses (1916), published by Longmans, Green and Co.

H. J. Ettlinger.

MATHEMATICAL ACCURACY

One hears frequent lay references to "mathematical proof" as an example of all that is rigorous and at the same time pedantic. There is the implication often slightingly suggested that the only rigor that one could anticipate for a subject of "human interest" is that evidence of decadence sometimes called rigor mortis. But mathematicians have been known to lapse from the steep path of logical exactitude, and natural sciences have always employed deductive reasoning in achieving their wealth of conclusions. While the theory of mathematics may illustrate more completely than any other line of human endeavor the possibilities to which deductive reasoning may be carried, there can be no excuse for supposing that the art of mathematics as practised is inherently less flexible than other arts. Just as in the case of morals, the philosopher might insist upon absolute lines of demarcation between the right and the wrong, while the jurist might emphasize the relative aspects of the problem, or vice versa, so in connection with mathematics one's general outlook will largely affect one's attitude toward the larger problems and their solutions.

We may perhaps expect every mathematical problem formulated today to be solved in some sense in the not too distant future, say within three centuries. This may be achieved in many cases by the discovery that we are today attempting an impossibility, that we are seeking a solution of a *particular form* when no such solution can exist. On the other hand, few important topics are so completely studied that no new light upon them can be looked for. Mathematics, like other subjects, is widening its scope. The distant frontiers now faintly discernible will doubtless be familiar landmarks in the future. But unlike the more stable geophysical features, the internal characteristics of the most traversed territories are liable to significant alteration as a result of widening our scientific boundaries. And, of course, the total boundary-land of unexplored but adjacent jungle is ever widening.

It is possible to regard every item in a mathematical computation as equally important. This is in fact the easy and natural thing to do. An error is an error and there need be no mitigating circumstances. When it is a question as to the steps in a proof, no such absolute standards are usually practicable. A strictly Euclidean formulation with its appeal to axiom, construction, and previous proposition, is so cumbersome as to be seldom desired. It is often therefore impossible to criticise a given proof as erroneous. We can sometimes merely state that the proof at least omits some steps apparently necessary for clarity, and leave to our imagination the question as to whether the original author could have filled in these vital lacunae. But even in algebraic work it is often desirable to distinguish between items of different degrees of importance. This is true particularly in the selection of physical problems as exercise material for a class. Is it better to make the problems more involved and at the same time give them a "practical" turn, or is it to be preferred that the principles under discussion appear distinctly, unhampered by mere arithmetical encumbrances and non-pertinent interpretations? This is an important question, and arguments for either side can not be set aside as trivial. Shall the student be drilled in combined application of many principles, or shall most of his time be better employed by directing his attention more forcibly than otherwise possible to individual difficulties, leaving for other courses or for review work only, their numerous possible combinations? Shall we cover much ground with relatively easy topics, or tackle the more difficult items at the risk of leaving much interesting and useful material untouched? At the start of the subject of logarithms, shall the first lessons emphasize the placing of the decimal point, or will interpolation be given primary consideration?

When one seeks to apply mathematics to practical problems, the whole question of approximations must be faced. Shall an infinite process be regarded as taboo merely because it is known to be theoretically infinite, or shall the

use of the first few terms when these are learned to afford a rapid approximation be encouraged? Shall we allow the suppression of "negligible terms," or must other methods of analysis be employed? The use of rates in elementary physics without direct appeal to the notions of differential calculus, results at times in wierd proofs obnoxious to the student on account of their apparently obvious scorn of rigor in neglecting "infinitesimals of higher order."

To insist that the pupil shall make no errors in arithmetic has its clear advantages in developing accuracy and unhesitating facility, but it has also its baneful reactions in encouraging him to regard all errors as of the same category and hence as equally excusable or even to condone those that are most easily made as least objectionable. In multiplying numbers expressed in the decimal notation many grade school pupils regard the misplacing of the decimal point as trivial. The confusion in the use of percentage whereby the factor, 100, is omitted is looked upon as of no serious import. Even teachers have been heard to explain that the method of determining what percentage 7 is of 11, is to divide 7 by 11 and express this as a decimal, the quotient so obtained being the percentage desired. In a problem in geometrical mensuration, involving the area of a circle, many pupils think that the error in confusing the length of the radius with that of the diameter is pardonable, while to use 3 for the value of Pi, would shock nearly all of them. But examples of this sort need not be multiplied.

The pupil is taught in school that 1 is the same as 1.000000...000, carried out as far as one may desire, and comes to regard this as self-evident. This is exactly contradictory to the practice of physicists. The meaning of $1.x10^6$ is very different from that of $1.00x10^6$. In the former case all that is meant is that a certain quantity has been estimated or observed to lie somewhere between 500,000, and 5,000,000, with 1,000,000 assumed to be a reasonable mean, in the latter case, the quantity is understood to lie between 950,000 and 1,050,000. It has been asserted that no direct physical measurement is so accurate as to

justify its expression to more than six significant figures. Whether this assertion be justified or not, it is certain that many measurements do not approximate the accuracy that is sometimes assumed. In many cases there is no immediate meaning as to what is the *exact* value. Thus one may remark that the sun is ninety-three million miles from the earth. We all know that this is expressed in round numbers. If we ask for the exact number of miles the question becomes complicated in many ways aside from the possible difficulties in observation. We must in such a case agree as to the points between which the distance is to be measured and the instant at which the measurement is to hold. The earth is not a point, nor is it an exact sphere. If we refer to its center, we must specify in what sense this is to be a center. The center of mass is probably not the center of shape as this term is often used and this again may not be the mid-point between the poles as usually determined, and so forth, and of course the sun presents even worse problems, seeing that even the term, "surface of the sun" has many distinct uses. The folly of attempting measurement except under known conditions of temperature, pressure and so forth is so well known that any unqualified statement of measurement in figures cannot be absolutely accurate but only valid to an assigned number of significant figures. The pupil in school seldom appreciates the basis for the figures that are given him. He sometimes comes to believe that all reasonable problems appear by nature with integral coefficients and usually have integral solutions. He seldom, if ever, appreciates that an approximate answer obtained quickly and without blunders is worth infinitely more than long but seriously erroneous results secured with much labor.

The fault for this situation does not lie wholly with pupil or teacher. Many books seem to demand an attention to the trivial while ignoring the essential that is unworthy of American institutions. Such problems as the following are not rare:

1. Find the cost of a carpet at 11.93 cents per square yard, which measures 40½″ by 93¼″.

[One might urge that except under the most unusual circumstances only oriental rugs could come in such odd dimensions, and oriental rugs are not priced by the yard! In one arithmetic text the answer to a problem like that above was given to the thousandth of a cent.]

2. Find the contents of a wooden box whose base measures $11\frac{1}{3}$ inches by $2\frac{1}{5}$ feet, and which is 17 inches high.

[Nothing is here said about the thickness of the boards used in making a box of these curious dimensions, nor is it known whether there be a cover that sets in, reducing the available free space to be computed. No carpenter will use thirds of an inch or fifths of a foot. The answer is given as a fraction to the nearest fifteenth of a cubic inch.]

3. Find the total volume of three-quarter inch by twelve inch planed pine lumber of which thirteen lengths each eighteen feet long are to be purchased.

[This sounds like a practical problem. Lumber of these dimensions may be indeed bought at a planing mill, and paid for upon the basis of this computation together with the labor cost involved in the planing. Even so the result worked out to the nearest cubic inch has only a fictitious meaning. Long planks are not cut off squarely to measure but come from the mill unless otherwise explicitly ordered, with several inches to spare at the end. The ends are not usually cut square so that the computation based on observed length is not so simple. Now every one conversant with mill practice will acknowledge that planed pine stock marked as of width, twelve inches, will actually measure after drying and planing something like eleven inches and similarly the actual thickness will not be over five-eighths of an inch.]

The student who will calmly assert that 23 is the number half way between 2.1 and 2.5, that three-eighths of a is expressible as $37\frac{1}{2}\ a$, or who requires paper and pencil and much time to estimate roughly the square root of 80, and to approximate the product of 10.2 by 9.9, may have received next to nothing of value from his arithmetical training.

There are many persons who can keep track of their personal accounts to the nearest cent but who regard it as obvious that it will be cheaper to take out the family auto, run down on a little two-mile spin to the headquarters of the ice company, pay twenty cents for a piece of ice, carry home what does not melt of this on the running board, and themselves place the remnant in the refrigerator, than to pay a quarter for the service of having it brought and put in for them by the employe of the company. To keep track of payments for the postage stamps and to have no ability to estimate depreciation, overhead costs, or other more serious financial items is to lose complete sight of all true notions of accuracy. Petty accuracy is indeed desirable and occasionally essential, but let us hope that neither teacher nor pupil will ever allow trivial dexterity with numbers to supplant that accurate weighing of essentials and emphasis upon the relatively larger numerical magnitudes that alone give value and dignity to the applications of arithmetic.

A. A. BENNETT.

THE HISTORY AND ELEMENTARY TEACHING OF NUMBERS

Teachers of freshman mathematics in college find their students have very vague ideas about all numbers other than fractions and integers. Perhaps one should say other than integers, for I have found students who add fractions by adding numerators and denominators or, in case the numerators are unity, by adding the denominators. Another case also will illustrate a lack of understanding of the fractional symbol. Some do not discriminate between $\frac{2/3}{5}$ and $\frac{2}{3/5}$ and are just as likely to multiply the numerator or the denominator by 5 in either case. Not only are the terms "real," "rational," "irrational" and "imaginary" confused, but the concepts themselves are not clear in the student's mind. One student relieved herself of the trouble of dealing with irrational roots of a quadratic by simply erasing the radical signs. Others relegate the irrational roots to non-existence, as did the ancients, and say there are no roots. The number 0 they accept too graciously and are ready to accord it all the privileges other numbers possess. To some extent these numbers must remain a mystery to the elementary students. For instance, it would be out of the question to present to them the Dedekind definition of an irrational number.

These mental reservations which the student feels about accepting negative, irrational and imaginary numbers we find displayed by the race itself during previous centuries. For this reason I believe a knowledge of the history of how these numbers appeared and finally were accepted would enlarge the view of the student. Symbols representing the positive integers appear in our earliest records. The simple process of counting offered little difficulty. The extension of the notion of number to that of rational integral fractions involves further the concept of magnitude. Symbols

for such fractions are found in the very early records of the Babylonians. Euclid considered commensurable ratios but did not regard them as numbers. The first actual use of rational fractions was made by Diophantus of Alexandria (300 A. D.). The earliest work on mathematics which has come down to us, the Egyptian papyrus of Ahmes (1700 B.C.) makes use of fractions with numerator unity.

The notion of an irrational quantity arose about the time of Euclid or some of the Greek philosophers preceding him in connection with the consideration of incommensurable lengths such as the side and diagonal of a square. However, these ratios were not at that time considered numbers. It was not until the end of the 16th century that the irrational numbers as such appeared, this time in Western Europe. At that time the decimal system of notation was beginning to be used for the representation of fractions. It was found that some fractions such as $\frac{1}{3}$ when so expressed led to a repeating infinite decimal. Naturally, then, the question of infinite decimals which do not repeat arose, and mathematicians began using them in the same way as other numbers without inquiring as to the propriety of such use. Not until the 19th century were they put on a scientific basis by Weierstrass, Dedekind, and others.

The notion of a negative number extends our concepts of number still farther to include that of direction. The first need for negative numbers arose in algebra in connection with the solution of equations. The first person to recognize negative roots was the Hindu, Bhaskhara (1150 A.D.). He obtains $x=50$ and $x=-5$ as roots of the quadratic $x^2-45x=250$ but says "the second value in this case is not to be taken for it is inadequate; people do not approve of negative roots." *Not for more than 300 years were they accepted. Cardan (1501-1576) speaks of a "pure minus," but not until the time of Descartes (1596-1650) was the use of negative numbers definitely established. Algebraically, the purpose of their introduction was to make subtraction possible between any two numbers.

*Cajori, "A History of Mathematics," p. 93.

Complex numbers appeared as early as the 16th century in Cardan's solution of the cubic equation. But mathematicians were loath to accept them as numbers and not until the 19th century were they admitted and their use established. After the introduction of negative numbers, the imaginary number was needed in order to make the extraction of roots always possible.

The number zero is a more unique member of our number system than most students think. There is no Roman symbol for zero. It arose with the Arabic notation in which a digit takes its value from its position in the number. Thus in 23, the two stands for two tens because it is at the left of the 3. In case they wanted to represent such a number as twenty, they had no symbol to place in the position of the number 3 in the former case and so they invented the symbol 0 to occupy that position. It was not then considered a number. As such, it probably was established, as were negative numbers, to make subtraction always possible.

This knowledge of the origin and growth of the idea of number will make the use seem less artificial to the student. Positive and negative real numbers are best represented as lengths measured from a point on a straight line, the positive in one direction and the negative in the other. At first the student is inclined to think the irrational numbers have no corresponding lengths, but by taking a few examples such as diagonals of squares he readily recognizes that there are such lengths. This mode of representation makes clearer what is meant by one number being "greater than" or "less than" another number. As to the carrying out of the algebraic processes with irrational and imaginary numbers, the ability to do so can only be acquired by considerable drill and practice after the methods have been explained.

JESSIE M. JACOBS.

The historical material in this paper was largely obtained from Young's "Fundamental Concepts of Mathematics."

JESSIE M. JACOBS.

CURVE TRACING IN ANALYTICAL GEOMETRY

If almost any one of our text-books in analytical geometry is followed closely, the student does a great deal of point plotting of equations. The beginner learns much by point plotting, and it is not my object to disparage it, but rather to encourage the teacher to build on it and acquaint the student early in his study of the subject with notions that ordinarily do not reach him until later in the course.

For plotting equations of higher degree than the first, it is necessary to have definite methods of study. It is to be remembered that the curves of analytical geometry are the loci of real points (points whose coördinates are real numbers) whose coördinates satisfy the equation of the curve, and that there are equations that have no real loci. The equation $x^2+2y^2+3=0$ is an equation of this kind. Many others might be cited. The reader is reminded also that he must be on the constant lookout for equations whose left members are factorable, since such equations when plotted are the combination of the plots of the several factors equated to zero. The equations $4x^2-9y^2=0$ and $x^2+xy-3x-8y+2=0$ are of this kind. It will now be indicated how familiarity with certain simple equations enables the student to recognize by the equations a large number of curves of elementary geometry.

For the sake of continuity in these remarks rather than feeling it is necessary, consider the equation $x^2+y^2=r^2$, a circle with the origin as center and with radius r. An equation of this type should be plotted only once, if at all, for it is so easy to prove the locus is a circle. It is readily seen that the more general equation of a circle $(x-\alpha)^2+(y-\beta)^2=r^2$ may be regarded as saying that points on the corresponding curve lie relative to $x-\alpha=0$ and $y-\beta=0$ as points on $x^2+y^2=r^2$ lie relative to $x=0$ and $y=0$. Equations of the type $x^2+y^2+Dx+Ey+F=0$, by algebraic manipulation being reducible to the above general form, are therefore in the circle category, and should never be point plotted.

Equations of the type $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, for example $4x^2+16y^2=64$, by considering symmetry, intercepts, excluded values, and plotting a few points, are found to be ovals, or, more exactly, ellipses. Furthermore, an equation of the type $Ax^2+By^2+Dx+Ey+F=0$, A and B like in sign, can be written in the form $\frac{(x-\alpha)^2}{a^2}+\frac{(y-B)^2}{b^2}=1$, and, using the notions indicated for the circle, may be regarded as an ellipse (or a circle in case A=B) lying relative to the lines $x-\alpha=0$ and $y-\beta=0$ as $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ lies relative to the axes of reference. Similar considerations show that an equation of the same type but with A and B unlike in sign is reducible to $\frac{(x-\alpha)^2}{a^2}-\frac{(y-\beta)^2}{b^2}=1$ or else to $-\frac{(x-\alpha)^2}{a^2}+\frac{(y-\beta)^2}{b^2}=1$ and is an hyperbola lying against the lines $x-\alpha=0$ and $y-\beta=0$ as the equation $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ or $-\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ lies against the axes of reference. It is to be emphasized that the few points that need to be plotted may be obtained as satisfying the simpler equation, and when thus obtained are to be plotted relative to the lines $x-\alpha=0$ and $y-\beta=0$ as axes of reference. The algebraic and arithmetic work involved in calculating the coördinates of points is greatly reduced by this scheme.

The equation next in simplicity and interest is the equation $y=ax^2$. By considering symmetry and intercepts and plotting a few points, the curve is found to be a parabola with the origin as its extreme point and with its arms opening up or down according to whether a is positive or negative. By a similar study, the equation $x=ay^2$ is found to be a parabola with the same extreme point but with its arms opening to the right or left according to whether a is posi-

tive or negative. With these equations well in mind, any equation of the type $y=ax^2+bx+c$ or of the type $x=ay^2+by+c$ can be plotted easily. For example, the equation $y=2x^2-3x+5$ can be rewritten in the form $y-\frac{31}{8}=2(x^2-\frac{3}{2}x+\frac{9}{16})$ and then in the form $y-\frac{31}{8}=2(x-\frac{3}{4})^2$. It is therefore a parabola lying relative to the lines $x-\frac{3}{4}=0$ and $y-\frac{31}{8}=0$ as the simpler curve $y=2x^2$ lies relative to the axes. The curve has its arms opening upward and has the point $(\frac{3}{4}, \frac{31}{8})$ as its extreme point (lowest point in this case). Here, as above, it is to be noticed that the few points that are plotted can be obtained as points satisfying the equation $y=x^2$ and when thus obtained they are to be plotted with reference to the lines $x-\frac{3}{4}=0$ and $y-\frac{31}{8}=0$.

The student becomes familiar with all that has been said thus far in connection with the translation of axes, but the notions are so simple, and such considerations are so helpful in plotting that it seems unnecessary as well as unpedagogical not to present them early in the course.

An equation of importance and interest is the equation $xy=k$, the equilateral hyperbola. For concreteness, consider the equation $xy=1$. By noting that there is symmetry with respect to the origin, that the second and fourth quadrants are excluded regions (since x and y must be like in sign), and that when x is large and positive y is small

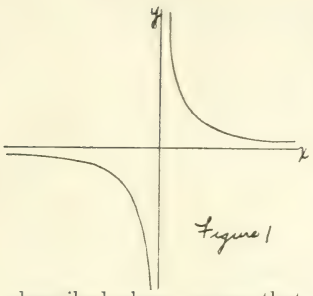

Figure 1

and positive, when x is large and negative, y is small and negative, when x is small and positive, y is large and positive, and when x is small and negative, y is large and negative, and finally obtaining a few points that satisfy the equation, it is found that the curve is the curve of Fig. 1. It is to be emphasized that the variation of x with y as described above means that the x and y axes are asymptotes to the curve and that the curve lies relative to these asymptotes as the figure indicates. It is desirable to remark here that to know certain lines are asymptotes and no more is of no particular help in plotting a curve. What is helpful is to know how a curve lies relative to its asymptotes. This is to be determined by considerations similar to the above. Building on the curve xy=1, the student can find for himself that the equations xy=2, xy=3, represent curves similar in shape and position to xy=1 and that the larger the right member, the farther the branches lie from the axes. The study of xy=k for k negative shows that curves of this kind lie in the second and fourth quadrants exactly as the curves xy=k lie in the first and second quadrants.

Now curves whose equations are reducible to the form (x—a) (y—b)=k are easily disposed of, and the writer feels it is to the joy of the student, for the college freshman finds these simple asymptote problems as presented in the usual textbook very difficult. With the above preliminary considerations in mind, it is readily seen that the equation (x—3) (y+4)=1, for example, is an equilateral hyperbola lying against the lines x—3=0 and y—4=0 as does xy=1 relative to the coördinate axes, and the curve can be traced by reproducing the curve xy=1 plotted above against the lines x—3=0 and y+4=0, as in Fig. 2. The type of equa-

tion so reducible is $xy+ax+by+c=0$. It can always be rewritten in the form $(x+b)(y+a)=(ab-c)$.

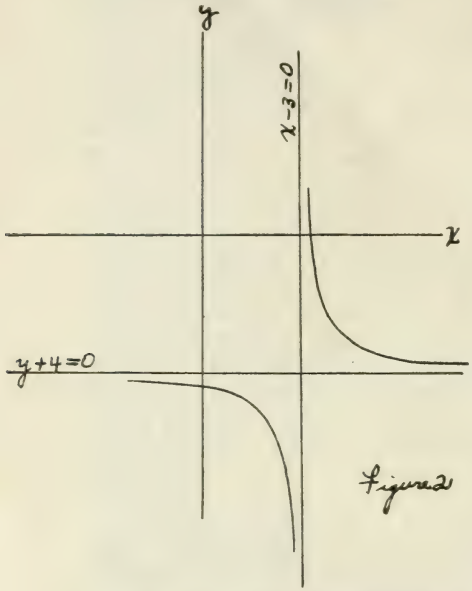

Figure 2

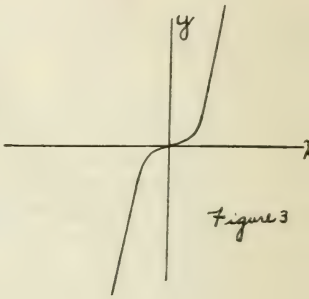

Figure 3

Two more simple equations will be mentioned. The type $y=ax^3$, the cubical parabola, is of interest. For simplicity, consider $y=x^3$. It is found to be the curve of Fig. 3. The curves $y=2x^3$, $y=3x^3$, . . . are found to be curves similarly placed. The equations $y=-x^3$, $y=-2x^3$, . . . should be studied, and also the equations $x=ay^3$, for a, both positive and negative. After studying these equations, the student is ready to plot any equation reducible to the form $y-b=a(x-c)^3$ or the form $x-b=a(y-c)^3$. Consider, for example, $y-1=(x+3)^3$. This curve lies relative to the lines $x+3=0$ and $y-1=0$ like $y=x^3$ lies against the axes, and it can be plotted by reproducing the curve of Fig. 3 against the lines $x+3=0$ and $y-1=0$, as indicated in Fig. 4.

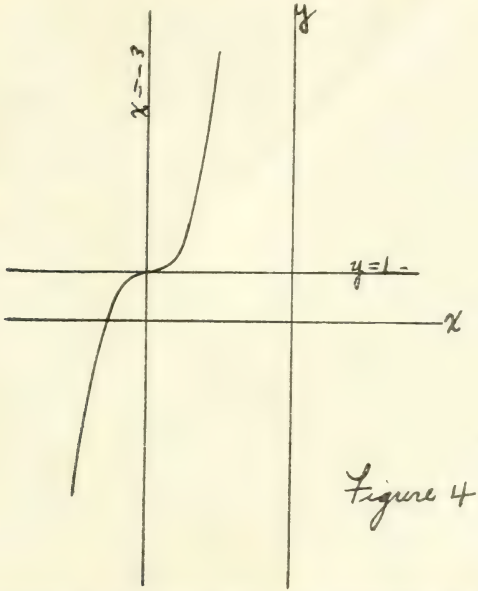

Figure 4

It should be noticed that an equation reducible to this type has a prescribed relation among its coefficients of x^3, x^2 and x. For example, the equation $y=2x^3-6x^2+6x-7$ is reducible to this type, while the equation $y=2x^3-6x^2-7x-7$ is not so reducible.

The semi-cubical parabola, $y^2=ax^3$, is another simple equation that might be studied in a similar manner. By a study of this equation and its mate, $x^2=ay^3$, equations like $(y-1)^2=3(x+2)^3$ and $(x+2)^2=5(y-1)^3$ can be plotted readily and with a minimum of labor.

In conclusion, a few remarks about the intersections of curves with lines and intuitive notions about tangency will be made. The student of analytical geometry learns quite early that setting the variable y equal to zero and finding the corresponding values of x gives the intersection of the curve with the axis of x, with equation $y=0$. He rarely

realizes, however, that setting y equal to any other value, say 2, and getting the corresponding values of x means that he is getting the intersections of the curve with the line y=2. It is a special case, of course, of setting y equal to any expression, say 2x—3. He is then finding the intersections of the curve with the line y=2x—3. And this, in turn, is a special case of the general problem of finding the intersections of two curves.

The student of elementary analysis finds interesting such intuitive notions of tangency and the evolution of peculiar points as will now be indicated briefly. Consider first the equation y=k(x—a)(x—b). When y=0, the values of x are a and b. That is, the curve crosses the x axis at (a, 0) and (b, 0), with its arms turned up or down according to whether k is positive or negative. Now let a and b close down on some value, say 2. The parabola crossing the x axis at two distinct points has evolved into a parabola touching the x axis at the point (2, 0). When y is zero now, x has the values 2 and 2. This appearance of a double value of x for a given value of y can be counted upon as meaning tangency to the line determined by the value of y. In an analogous manner, the student can be shown that if a, b, c of the equation y=k(x—a)(x—b)(x—c) close down upon some value, say 2, the resulting curve both crosses and touches the line y=0 at the point (2, 0). That is, the point (2, 0) is a *point of inflection* on the curve. This appearance of a triple value of one variable corresponding to a given value of the other can be counted upon as meaning a point of inflection. A similar study of the equation y=k(x—a)(x—b)(x—c)(x—d) will show that when the a, b, c, d close in upon some value, say 0, the curve evolves into $y=kx^4$, that in general appearance it is like $y=kx^2$, but that it has what is spoken of as contact with the x axis of higher order than has $y=kx^2$. These notions may be extended. They find place in the plotting of curves and are most helpful to the student in interpreting his algebraic results.

GOLDIE P. HORTON.

GEOMETRY IN THE SEVENTH GRADE

For more than a century the question of a course in introductory geometry has been debated by many of the best educators. The result is that now there is a general agreement as to the practical value and desirability of this subject. The International Commission on the Teaching of Mathematics has collected and published the mathematical curricula, both primary and secondary, of the principal civilized countries. *From a study of this curricula it appears that the United States is the only country which thus far has not recognized the importance of geometry for young pupils.

In England instruction in geometry is begun in the third school year and continued in some form for six years. No other foreign countries postpone geometry beyond the seventh school year, and in most cases the work is begun in the fifth school year and continues for a number of years. The work of the fifth year in most European countries is decidedly more extensive than in most of the schools in the United States. Probably the most marked difference between the fifth grade work here and abroad is the large amount of time and attention put upon the propaedeutic study of geometry.

The European countries give all of the arithmetic that we give to our pupils, together with an extensive study of geometry. The question arises, has America worked out a curricula peculiarly suited to American countries? It is our custom to limit the young pupils to the rudiments of mensuration, taught in connection with arithmetic, without any attempt at a systematic development of form study. On such an inadequate foundation we super-impose the formalism of algebra which has little meaning to the immature student. We next take up demonstrative geometry. We expect the pupil to assimilate a multitude of definitions,

*Brown, J. C., Curricula in Mathematics, 1915. Washington, Government Printing Office.

ideas, and processes, in a few weeks, not to mention the difficulties of logic, which he is totally unprepared to appreciate. Many of our mathematicians and leading educators believe this arrangement to be all wrong. Instead, some work in geometry of an introductory nature should precede algebra.†

Being thoroughly in accord with this idea, we have introduced, in the seventh grade, intuitive geometry. It is given in the high seventh grade and could be used in the eighth grade as advantageously; but in our program we offer elementary algebra in the low eighth grade. This work, however, should always precede algebra. There might be those who would oppose a course in intuitive or observational geometry in the seventh grade on the supposition that it would eliminate some of the work in arithmetic and would weaken the pupils in mathematical ability. Such is not the case, but additional topics in arithmetic are included. A course of this kind is closely correlated with arithmetic. This is as it should be, for arithmetic and geometry have grown side by side, and in the beginning geometry was a thoroughly practical subject until the Greek philosophers added so much logic to it.

Why give geometry in the seventh grade? Before we attempt to answer this question, we shall first ask, what do we expect the first six grades to accomplish? We should say the following: ability to count, read, and write numbers; accuracy and rapidity in the four fundamental processes operating in the field of whole numbers and fractions; simple exercises in the application of the processes to everyday problems; a knowledge of the tables of measure in common use, ability to reduce such tables; and, lastly, general information concerning business practices. Whatever is given in the first six years beyond these essentials consumes time that might well be spent in other ways. While this enumeration suggests generous omissions from our textbook, it also proposes that better results be obtained within

†"Psychology of High School Subjects," by C. H. Judd, Ginn & Co., 1915, pp. 20-22.

the limits than are usually secured in our schools, particularly the acquiring of accuracy and facility in the fundamental processes. The opinion that the mastery of these mechanical processes demands a long period of years is erroneous. Not alone from the standpoint of those who drop out of school at the end of the sixth year should these results be obtained, but they are needed as well by those who go on. The average age at which our pupils complete the sixth grade work is twelve years. The adolescent boy or girl of twelve has a very inquisitive and investigating mind. During this transition period pupils should be led to rationalize and generalize. Mathematics should be presented as a thoroughly practical subject and an instrument of use. It is well at this age, or even earlier if possible, to introduce the pupil to this universe of form. The study of form in nature, in architecture, or in his own environment increases his aesthetic enjoyment and his usefulness in many careers. A study of this kind implies constant emphasis on directed observation. Then the pupil must be led to describe these forms in his own language. Gradually, he will acquire a suitable mathematical vocabulary and permanent mental images of geometric forms. Observation, description, drawing, and modeling are the chief means of securing that result. By measuring and drawing their own figures with geometric instruments, the pupils become familiar with the fundamental ideas of equality, symmetry, congruence, and similarity, which prepare them for further study. In a course of observational geometry for boys and girls of this age, they should be encouraged to do their own exploring. With very little guidance, the pupil soon discovers important geometric truths. His curiosity leads him to seek new truths and ask the reason. The study of geometric forms and the underlying principles, combined with numerous applications, will not only make this subject extremely valuable in itself, but will also make it an indispensible foundation for later study of algebra and formal geometry. The work in mensuration and its application by the use of the formula is the best presentation of algebraic symbolism.

The first few lessons in elementary geometry should be largely historical in character. They might well include a short history of the early methods of counting and number systems. Pupils enjoy this work. It is "different." They are tired of such a sameness in the arithmetic work. Creating a new interest in the mathematics work just at the time when the boys and girls are desiring it gives a good start to your course. Every good teacher of mathematics should be well acquainted with the history of his subject. Without the history of mathematics it is impossible to understand the relative significance of its principles and applications. The fact that historical information about mathematics is interesting and helpful to children explains to us why geometry in the senior high school is so often not a success. Teachers do not have time to bring this in, and it takes this foundation to get the proper interest and enthuiasm in the study of geomtry. If we are permitted to put intuitive geometry in the junior high school or seventh and eighth grades, the origin of mathematics should be the basis of the course.‡

The use of geometric instruments should be emphasized. Each pupil should be provided with the necessary drawing instruments, such as, ruler, compass, and protractor. The ruler should be ruled in both English and metric units. In our school, the Allan Junior High School, each pupil has a notebook containing good white paper and a few sheets of squared paper which are needed for drawing designs and graphs. The work is developed on a laboratory basis. Each new principle is introduced by means of concrete exercises. The main question to be asked is, what do you see, or what do you think? The pupils enter their observations

‡We are indebted to Mr. Wm. Betz, Head of the Mathematics Department, and Vice-Principal of the East High School, Rochester, New York, for helpful suggestions in this work. In regard to his own work he says, "Geometry has been taught in the Junior High Schools of Rochester since 1915 in the eighth grade, and since 1920 in the seventh grade. This year it has been introduced into all city schools of Rochester in the seventh grade, and is continued in some form until the tenth school year."

and conclusions in their notebooks. Thus, during this study many important theorems in geometry are discovered by observation and measuring. The pupils come into possession of the facts intuitively, but no formal proof is attempted. The recitation is very social in character. The pupils are working and learning, with the teacher in the background occasionally explaining or asking questions to lead the pupils on in their discoveries.

In planning a course of this kind, a choice and sequence of topics which seem to be adapted to the transition period should be kept in mind. From our own experience with the work, we suggest a few topics that are within the ability of the pupils of this age and have proved to be interesting to them. The intuitive geometry course should comprise the rudiments of geometry and include classification of form, preliminary designs, followed by a systematic study of lines, circles, angles, triangles, polygons, parallels, symmetry, congruence, and the areas of geometric figures, both simple and complex. The new topics in arithmetic which are brought into use with the geometry are the metric system, changing from one unit to the other, square root and its applications, ratio and proportion, and many practical applications of designs; a study of the history and development of the Pythagorean Theorem, history of Pi (π), and practical problems showing the use of both. Mensuration should include a study and development of the formulas for finding areas and volumes of such figures as the prism, pyramid, cone and sphere. Each formula should be deduced from some previous knowledge of geometric form and the rule not simply handed out. The mensurational power of geometry enters in so many ways into the world's ordinary affairs that any boy or girl ignorant of the first principles of geometry would be handicapped from a standpoint both of general culture and of life preparation. By our giving this work in the seventh grade, the student who is obliged to drop out of school and go to work has an opportunity to secure this necessary preparation. Our course in intuitive geometry also should include simple geometric constructions, simple cases

of loci and graphs such as bargraphs, circular graphs and curves.

In the upper grades in the high school it is hard to bring the various branches of mathematics together, and they are often taught by themselves. In the above outline of a course in intuitive geometry it is possible to correlate arithmetic, algebra and geometry. After a geometric foundation has been laid in the lower grades, extensive correlation of the different subjects should be carried out wherever possible. Every possible opportunity for correlating the high school subjects should be taken advantage of, for this is, in our opinion, preferable to a fusion of the subjects. It is very evident that it is the problem of the mathematics teachers of today to enliven the subject matter of our curricula, to add interest, and to create a desire for more mathematical knowledge on the part of the students. Observational geometry can be made a new field of interest to the pupils, for it affords laboratory work, and many mathematical projects may be undertaken. It is undeniable that the old high school curriculum has failed in creating insight or motive by not securing a permanent grasp of principles, and ignoring, through lack of time, the most obvious application. All of these neglected features must be cultivated in the lower grades and a beginning can be made in intuitive geometry.

In conclusion, we should like to say that we have been very much pleased with our year's experience with the work. The pupils have enjoyed it and have shown great interest. We feel that they have been benefitted greatly. The percent passing in the work has been above the average. The "ne'er-do-wells" in other subjects were likewise our failures in intuitive geometry. We should be glad to see more done in this line.

Briefly stated, the aims of a course in intuitive geometry are to extend the pupil's knowledge in arithmetic through its practical applications in mensuration; to train the hand to use the simple drawing instruments; to familiarize the pupil with common geometric form; to train him to see

geometric form in nature and in the various buildings in his surroundings and to appreciate its use in design; through experiment and observation, to develop the formulas of mensuration; through continued study of formulas, to introduce general number in a natural way that will give algebraic expression such a meaning to the pupil that he will use it as a convenient and practical tool. In short, we aim to permit the pupils to live so continuously in the atmosphere of geometry that they may be enabled to think naturally and without confusion in its terms and about its relations.

Mrs. W. E. Odom,
Head of Mathematics Department,
Allan Junior High School,
Austin, Texas.

Mrs. Odom has been teaching this course during the past year and has the material ready now for a textbook which she will publish in the near future.

The Editors.

A REPORT OF THE MEETINGS OF THE REGULAR MONTHLY INSTITUTE OF THE AUSTIN PUBLIC SCHOOLS FOR THE YEAR 1922-1923

The teachers of the Austin public schools meet in regular session once each month for institute work. The first hour of each meeting is set aside for section meetings to enable the teachers to discuss their particular problems in group discussion.

The mathematics high school section is composed of the members of the mathematics departments of the Austin Senior High School and of the John T. Allan Junior High School. The chairman for the past year has been Mrs. W. E. Odom, head of the mathematics department of the John T. Allan Junior High School. Mrs. Odom, together with the assistant chairman, has had charge of the arrangement of the programs for the year. During this year the programs might be classified under two heads: first, reviews of mathematical articles taken from various copies of mathematical magazines; second, addresses from members of the faculty of the mathematics department of the University of Texas, a privilege which the Austin teachers fully appreciate.

During this year we have reviewed a number of articles taken from the various mathematical magazines. Some of the subjects are as follows: the teaching of mathematics, the need and the method; the psychology of errors in algebra; the fundamental principles of algebra; the grouping of pupils according to their ability; and the study of the quadratic equation. After the review of the article a round table discussion follows.

On March 10, 1923, the section had the pleasure of hearing Miss Mary E. Decherd, instructor in pure mathematics in the University of Texas. Miss Decherd talked to us on the subject "What Preparation We as University Instructors Expect." She limited her remarks to the subjects taught in the senior high school—algebra, geometry, and trigonometry. She covered the subject matter in a clear, con-

cise manner by presenting it in outline form. Many helpful suggestions on the presentation of subject matter were given, and the teachers felt free to ask questions on the particular phase of work in which they were interested. Miss Decherd's remarks were helpful to everyone present, but the teachers of the senior high school had an unusual interest because of the intimate contact of their work with the University.

On March 31, 1923, Dr. A. A. Bennett, chairman of the school of mathematics of the University of Texas, addressed the members of the section on the subject of "General Mathematics." He based his remarks on five claims for general mathematics and discussed these claims from the following terms: general, in that it enables the student to get a broader viewpoint; logical, in that it is more consistent to teach mathematics as a unit; practical, in that the student is getting a relation that he can better assimilate; novel, in that the mixing of algebra, arithmetic, and geometry is new to the student and instructor; simple, in that it makes the subject in general more easy to grasp. After the address, Dr. Bennett answered many questions in regard to the most natural place to fit in a course of general mathematics, and the subject best adapted for such a correlation.

The duration of the section meeting is one hour. At the close of the hour the teachers adjourn to meet with the entire body of Austin teachers for the remainder of the institute program.

MRS. GRACE R. WEST, Secretary.

Reports of meetings of other institutes over the state will be very acceptable for publication in the *Bulletin*.

The Editors.

Each of us will be a better teacher or a worse one next year than this. The summer will determine the matter.

In our rapidly changing world, new applications of mathematics are constantly arising. Are you keeping up with the procession or do you still think it is A.D. 1910?

The only laboratory equipment necessary for the study of the most accurate of all sciences is an alert mind and a willingness to use it.

Of course, students have other obstacles to overcome, but a teacher who does not study his lesson is one of the commonest hindrances to progress.

Why is it that most college freshmen when asked to define a right angle, say that it is an angle containing ninety degrees or that it is an angle formed by perpendicular lines?

The "cube of five" is not "five multiplied by itself three times." The result of multiplying five by itself three times would be three twenty-fives.

Professor Toot-Ah-Ho-Hum still finds that his students go to sleep in his class and wonders why they have no interest in mathematics.

Mathematics is universal.

The Bible has a Book of Numbers.

The Poet "lisps in numbers."

King Arthur conducted a "Round Table."

The politician conducts his campaign on a high (?) "plane."

The criminal develops a "line" of defense.

The poker player wants a "square" deal.

The wooer makes his girl a "proposition."

She tells him to see father and "axiom."

Wooer is "eliminated."